FEM-DBCI Solution of Open-Boundary Electrostatic Problem in the Presence of Floating Potential Conductors

G. Aiello, S. Alfonzetti, S. A. Rizzo, N. Salerno

Dipartimento di Ingegneria Elettrica, Elettronica e Informatica, Università di Catania, Italy

This paper extends the hybrid FEM-DBCI method for the solution of open-boundary electrostatic problems to case in which some floating potential conductors are present in the system. The iterative solution scheme of the standard method is modified in order to deal with the unknown values of the potential of these conductors.

***Index Terms*— Electrostatics, Finite element methods, Integral equations, Floating potential conductors**

I. Introduction

THE FEM-DBCI, is a hybrid numerical method devised by the authors to deal with static and quasi-static electromagnetic field problems in unbounded domains, such as electrostatic [1], [2], time-harmonic skin effect [3], [4] and eddy current [5] problems.

Similarly to the well known FEM-BEM (Finite Element Method - Boundary Element Method) [6-8], FEM-DBCI (Dirichlet Boundary Condition Iteration) couples a differential equation for the interior problem with an integral one, which expresses the unknown Dirichlet condition on the truncation boundary and involves free-space Green function. Differently from FEM-BEM, the FEM-DBCI integrals have a support strictly disjoint from the truncation boundary, so that singularities are avoided.

The resulting hybrid global system is solved by a simple iterative procedure: assuming an initial guess for the Dirichlet condition on the truncation boundary, the sparse FEM equation is solved by means of a standard solver for bounded problems, e.g. the conjugate gradient (CG) solver; the dense DBCI equation is then used to improve the Dirichlet condition [1]; the procedure is iterated until convergence is reached. This solution strategy is efficient because the CG solver is applied to the sparse equation and the dense equation is used only a few times by a fast matrix-by-vector multiplication.

In this paper the FEM-DBCI method is extended to the case in which some floating potential conductors are present in the electrostatic system. The method incorporates in the DBCI iterations the ones relative to the floating potentials.

II. FEM-DBCI for Floating Conductors

Consider the electrostatic system in Fig. 1 constituted by dielectric bodies, charge distributions and conductors embedded in an unbounded dielectric medium (free space). Some conductors are voltaged at given potential values V_k, $k=1,..,N_C$, with respect to infinity where the potential is assumed to be zero. The other conductors have assigned total charges Q_h, $h=1,\dots N_F$ (floating potential conductors).

In order to compute the electric potential v(x,y,z), a fictitious truncation boundary Γ_T is introduced. This boundary must include all the conductors and non-homogeneities, but it may leave out some (lumped or distributed) charges.

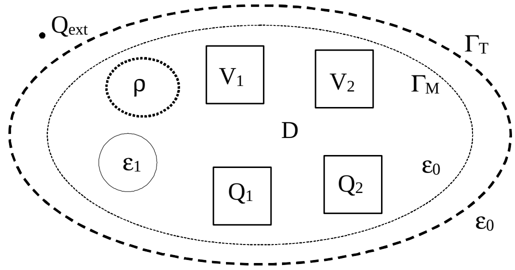

Fig. 1. An electrostatic system made of some voltaged or floating conductors, non-homogeneities and distributed charges, enclosed by a truncation boundary.

In the bounded domain D so obtained, the Poisson equation holds:

$$-\varepsilon_0 \nabla \cdot (\varepsilon_r \nabla v) = \rho \qquad (1)$$

where ε_0 is the vacuum electric permittivity, ε_r is the relative permittivity and ρ is the charge density. Dirichlet boundary conditions hold on the surface of the voltaged conductors, whereas unknown Dirichlet conditions are assumed to hold on Γ_T and on the surfaces of the floating potential conductors.

Discretizing the domain D by means of simplex nodal finite elements of a given order, the following FEM algebraic system is derived:

$$\mathbf{A}\mathbf{v} = \mathbf{b}_0 - \mathbf{A}_T\mathbf{v}_T - \mathbf{A}_F\mathbf{v}_F \qquad (2)$$

where $\mathbf{v}$, $\mathbf{v}_T$ and $\mathbf{v}_F$ are the column vectors of the potential values at the N internal nodes, at the N_T boundary nodes and at the surfaces of the N_F floating potential conductors, respectively, $\mathbf{b}_0$ is a column vector which constitutes the known term of the system and is due to the internal sources and to the voltaged conductors, $\mathbf{A}$, $\mathbf{A}_T$, $\mathbf{A}_F$ are sparse matrices of geometrical and constitutive coefficients.

The unknown Dirichlet condition on the fictitious boundary is expressed through:

$$v(P) = v_{ext}(P) + \int_{\Gamma_M} \left(v(P') \frac{\partial G(P,P')}{\partial n'} - G(P,P') \frac{\partial v(P')}{\partial n'} \right) dS' \qquad (2)$$

where Γ_M is a closed surface enclosing all the conductors and dielectric non-homogeneities, but strictly enclosed by Γ_T, $\mathbf{n}'$ is its outward normal unit vector (see Fig. 1), and G is the free-space Green function, given by

$$G(P,P') = 1/4\pi r \quad (3)$$

where r is the distance between points P and P', lying on Γ_T and Γ_M, respectively. In the numerical approximations this equation reads:

$$\mathbf{v}_T = \mathbf{v}_{ext} + \mathbf{H}\mathbf{v} + \mathbf{K}\mathbf{v}_F \quad (4)$$

where $\mathbf{v}_{ext}$ is a vector due to the external sources, **H** is a dense rectangular matrix in which non null entries appear for the nodes of the elements adjacent externally to the surface Γ_M and **K** is a rectangular dense matrix.

For the unknown floating potentials, it is necessary to write the following integral equations:

$$-\varepsilon_0 \int_{\Gamma_h} \varepsilon_r \frac{\partial v}{\partial n} dS = Q_h \qquad h=1,\dots,N_F \quad (5)$$

where Γ_h is the surface of the h-th floating conductors. In the numerical approximation this equation reads:

$$\mathbf{D}\mathbf{v}_F = \mathbf{q} + \mathbf{C}\mathbf{v} \quad (6)$$

where $\mathbf{q} = [Q_1\ Q_2 \dots Q_{N_F}]_t$ is the column vector of the charges in the floating potential conductors, **D** is a diagonal matrix and **C** is a dense rectangular matrix in which non-null entries appear only for the nodes of the elements adjacent to the surfaces Γ_h.

The global algebraic system is constituted by equations (2), (4) and (6). This system can be solved by means of an iterative approach, similar to that of the basic FEM-DBCI method:

1) assume initial guesses for the Dirichlet boundary condition $\mathbf{v}_T$ on Γ_T (e.g. $\mathbf{v}_T=\mathbf{0}$) and for the floating potentials $\mathbf{v}_F$;
2) solve the interior FEM problem (2), for example, by means of the CG solver;
3) use equation (4) to obtain another guess for $\mathbf{v}_T$;
4) another guess for the floating potentials $\mathbf{v}_F$ is obtained by means of (6), rewritten as:

$$\mathbf{v}_F = \mathbf{D}^{-1}\mathbf{q} + \mathbf{D}^{-1}\mathbf{C}\mathbf{v} \quad (7)$$

5) repeat steps 2, 3 and 4 until convergence is reached.

III. A Numerical Example

In this section an example is provided concerning an electrostatic system, constituted by a discharged square capacitor (thickness t = 1 cm, edge length l = 10 t, relative permeability of the dielectric ε_r=9) in the presence of a lumped charge Q_{ext} = 1.0 10^{-9} C, placed at a distance d = 5 t from the center of the upper armature of the capacitor, as shown in Fig. 2. Due to symmetry reasons, the analysis is restricted to a quarter of the system, by imposing homogeneous Neumann conditions on the xz and yz planes. The truncation boundary Γ_T is selected as a parallelepiped, placed at a distance d_T = t from the capacitor surface; the integration surface Γ_M has the same shape of Γ_F at a distance of d_F = t/4 from the capacitor.

The domain is regularly discretized by means of 34560 second-order tetrahedra and 93025 nodes, of which N_T=1789 lie on the truncation boundary and 882 on the two floating armatures. Having set an end-iteration tolerance of 10^{-4} per

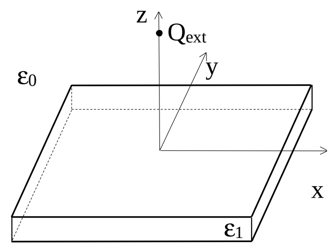

Fig. 2. A discharged capacitor in the presence of a lumped charge Q_{ext}.

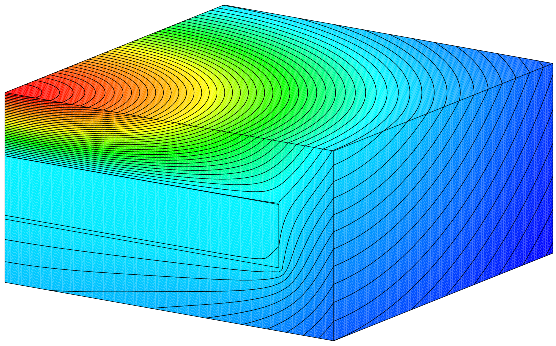

Fig. 3. Contours of the potential.

cent for the CG solver and 10^{-2} for the DBCI, convergence is obtained with 13 iterations. The CPU time is 5.73 s on an AMD Turion 64 X2 Mobile Tech. TL-64, 2.2 GHz, 2 GB RAM, GNU/Linux 64bit. Fig. 3 shows the contours of the electrical potential in the xz plane and on Γ_T.

Details and other results will be provided in the full paper.

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